Control and estimation for mobile sensor-target problems with distance-dependent noise

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***Abstract*— This paper investigates the scenario where a mobile sensor must observe (i.e., estimate the state of) another system, called the target. The estimation is affected by a distance-dependent noise, and for this reason we propose to control the sensor so that the effect of the noise is minimized. We propose a novel approach in which the controller and the observer are designed in tandem, with the common objective of obtaining a better estimation. We give sufficient design conditions for a general class of nonlinear systems satisfying a Lipschitz-like condition on the nonlinearity. A numerical example illustrates the obtained results.**

I. Introduction

We consider a scenario where a dynamical system, called the *target*, must be monitored – i.e., its state must be estimated – by a second dynamical system, which we call the *(mobile) sensor*. Examples include two unmanned aerial vehicles (drones), one of which is following the other, or two subsequent road vehicles in an automated platooning system. The challenge here is that the mobile sensor can only measure the output of the target with an error that is related to the distance between the two systems. This is typical when e.g. the position and attitude of an object must be estimated from video images, or with time-of-flight distance sensors as in e.g. [1], [2].

In order to accurately recover the target states, it is therefore important not just to design a good estimator, but also to control the sensor to reduce the error affecting the measurements. In this paper, we provide an approach that co-designs the controller and observer using Linear Matrix Inequality (LMI) design tools, which are commonly used in many areas of control, see e.g. [3], [4].

To explicitly provide an observer and controller design procedure, we focus on a specific class of systems by using the idea of a nonlinear observer design approach described in [5] and [6]. We have two objectives here: an estimation objective of accurately recovering the target states, and a control objective of moving the sensor system so that the target is tracked and that the estimation objective is achieved. For this problem, we first present (i) a monolithic approach, which defines the sufficient conditions for the error system to achieve the objectives. This method provides a design in terms of Bilinear Matrix Inequalities (BMI), which are difficult to solve. For this reason, we propose a second approach (ii) where we decouple the error system into an estimation error and a tracking error. With this decoupling the design becomes more conservative, but the conditions are defined in terms of LMIs, which can be efficiently solved. For both (i) and (ii) above, the target inputs are assumed known, which may not be always the case. Therefore, we finally consider (iii) the case when the target inputs are incompletely known, where the unknown part is modeled as a disturbance, and give design conditions that guarantee H_∞ disturbance attenuation [7].

We evaluate all three approaches (i)-(iii) in numerical experiments on two road vehicles, with dynamics taken from [8]. First, we consider the case when the sensor vehicle should reach the target, i.e., both should attain the same speed and position asymptotically. This might lead to collisions in practice, so with a change of variables we add a safe tracking distance between the two vehicles for collision avoidance. This collision avoidance procedure can be useful for more than just the vehicle dynamics in our example.

The framework we consider, where the sensor is dynamic and is controlled to actively reduce the observation error, is nonstandard in systems and control, so research in this area is not extensive. For example, in [9] a target A at an unknown location is considered and a mobile sensor B must move to the vicinity of A and then circumnavigate it. Inaccurate measurements of the distance between A and B are used to estimate the location of A. Compared to [9], our approach closes the loop on this distance-dependent noise, i.e. the mobile sensor is controlled so that this perturbation is attenuated, which is not an objective in [9]. Another example can be found in [10], where a nonlinear observer-based controller is applied to the problem of anaerobic digestion. In [10] the observer can be designed separately, without the control, but in our case the controller-observer problem must be solved in tandem so that the controller assists the estimation task.

Similar ideas have been more thoroughly explored in robotics, where the framework is called active sensing or active perception, see for instance [11], [12], [13], [14]. However, approaches in robotics largely do not provide stability guarantees, see e.g. [11], [14], and are often heuristically developed for specific problems, for example in [12]. In contrast, here we take a control-theoretic perspective, so our major interest is to provide analytical guarantees for our control and estimation problem.

In the sequel, following some definitions, we describe the problem statement in Section II. Then, in Section III

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we provide the monolithic and the decoupled approaches to solve the problem together with the results on disturbance attenuation. Numerical results are given in Section IV, and Section V concludes the paper.

Notation. Let $F = F^T \in \mathbb{R}^{n\times n}$ be a real symmetric matrix. $F > 0$ and $F < 0$ means that F is positive definite or negative definite, respectively. I denotes the identity matrix and 0 the zero matrix of appropriate dimensions. The symbol $*$ in a matrix indicates a transposed quantity in the symmetric position, for instance $\begin{pmatrix} P & * \\ A & P \end{pmatrix} = \begin{pmatrix} P & A^T \\ A & P \end{pmatrix}$, and $A + * = A + A^T$. The notation $\mathrm{diag}(f_1, ..., f_n)$, where $f_1, ..., f_n \in \mathbb{R}$, stands for the diagonal matrix, whose diagonal components are $f_1, ..., f_n$. Notation $\|s\|$ denotes the Euclidean norm of $s \in \mathbb{R}^n$. The notation $\langle \nabla V(x), f(x) \rangle$ refers to $\frac{\partial V(x)}{\partial x} f(x)$.

II. Problem statement

In this section we define the form considered for the target dynamics, the observer structure used and the control law. Based on these elements, the estimation and the tracking error are also defined for further analysis.

A. System description and assumptions

The target is modeled as:

$$\begin{aligned} \dot{x}_T =& A x_T + G\psi(Hx_T) + B(u_T + d) \\ y_T =& C x_T + D\omega(x_T, x_S) \end{aligned} \tag{1}$$

where $x_T \in \mathbb{R}^{n_x}$ represents the state, $u_T \in \mathbb{R}^{n_u}$ stands for the control input, $y_T \in \mathbb{R}^{n_y}$ is the measurement, $d \in \mathbb{R}^{n_u}$ is a disturbance and ω will be defined later. The nominal input u_T is known, but it is affected by a disturbance. This makes sense in many applications. For instance, in highway driving, a constant speed is usually maintained with some positive and negative accelerations, which can be considered as disturbances on top of a constant input. For another example, consider two drones that must execute a mission cooperatively while maintaining radio silence (e.g. for concealment reasons). The sensor drone is able to obtain the mission plan of the target drone in the beginning, and perhaps sporadically updated plans during the mission (so nominal inputs are known), but in-between, state estimation from passive sensors like camera images must be performed. Of course, due to unpredictable conditions in the field the inputs will change compared to the nominal ones, and these changes can be modeled using disturbance d.

The term $\psi(Hx_T) \in \mathbb{R}^r$ is an r-dimensional vector where each entry is a function of a linear combination of the states.

$$\psi_i = \psi_i \left(\textstyle\sum_{j=1}^{n_x} H_{ij} x_{Tj} \right), \quad i = 1, ..., r.$$

The vector ψ is a way of allowing nonlinearity in the dynamics, and must fulfill the following assumption.

Assumption 1: For any $i \in \{1, ..., r\}$ there exist positive constants $a_i \leq b_i$ bounded, so that

$$a_i \leq \frac{\psi_i(v) - \psi_i(w)}{v - w} \leq b_i, \quad \forall v, w \in \mathbb{R}, v \neq w. \tag{2}$$

□

Assumption 1 intuitively bounds the rate of change of the nonlinearity, and is a global Lipschitz property of ψ. Such an assumption is made e.g. in [15], [5], [6], [10]. In view of (2), it holds that

$$\psi_i(v) - \psi_i(w) = \delta_i(t)(v - w), \tag{3}$$

where functions $\delta_i(t) \in [a_i, b_i]$ for $i = \overline{1, r}$. The matrix containing these functions on the diagonal is denoted with $\delta_q = \mathrm{diag}(\delta_{q1}, ...\delta_{qr})$.

The output (measurement) equation in (1) is given by a linear term, Cx_T, perturbed by the unknown term $\omega(x_T, x_S)$, which is assumed to verify the next property.

Assumption 2: For any x_T, $x_S \in \mathbb{R}^n$,

$$\|\omega(x_T, x_S)\| \leq \max\{\zeta_1 \|x_T - x_S\|^{\zeta_2} - \zeta_3, 0\}, \tag{4}$$

where $x_S \in \mathbb{R}^n$ is the state of the sensor system, and $\zeta_3 \geq 0$, $\zeta_1 > 0$, and $\zeta_2 > 1$ are constants. □

Equation (4) implies that ω vanishes when the sensor state x_S is a ball centered at x_T of radius depending on ζ_3. In other words, the sensor is accurate when its states are close to that of the target. Assumption 2 is for instance verified by the model of [1] for radio-frequency (RF) communication, where the power ζ_2 is the path loss exponent, see also Section III-D.

It is assumed that the sensor model is the same as the target model: it has the same dynamics with the same number of states and inputs

$$\dot{x}_S = A x_S + G\psi(Hx_S) + B u_S, \tag{5}$$

where $u_S \in \mathbb{R}^{n_u}$ is the control input of the sensor system to be designed, and all the states of the sensor system, x_S, are available.

We consider the following structure for the observer:

$$\begin{aligned} \dot{\hat{x}}_T &= A\hat{x}_T + G\psi(H\hat{x}_T) + Bu_T + L\left(y_T - \hat{y}_T\right) \\ \hat{y}_T &= C\hat{x}_T, \end{aligned} \tag{6}$$

where $\hat{x}_T \in \mathbb{R}^{n_x}$ is the estimate of x_T and L is the observer gain. In the observer design the nominal input of the target system, u_T, is considered as a known term, but not the disturbance, d.

B. Error dynamics

To state the problem, we introduce the estimation error, $e := x_T - \hat{x}_T$, and the tracking error, $z := x_T - x_S$. Since we do not have direct access to z, we use its estimated version, $\hat{z} := \hat{x}_T - x_S$, which can be rewritten as $\hat{z} = z - e$. Now we are ready to define the estimation error dynamics:

$$\begin{aligned} \dot{e} =& (A - LC)e - LD\omega(x_T, x_S) + Bd \\ &+ G\left[\psi(Hx_T) - \psi(H\hat{x}_T)\right]. \end{aligned} \tag{7}$$

On the other hand, the tracking error dynamics is

$$\dot{z} = Az + G\left[\psi(Hx_T) - \psi(Hx_S)\right] + B\left[u_T + d - u_S\right] \tag{8}$$

Since the estimation error is affected by the distance between the sensor and the target through the unknown term ω, we use a control law that moves the mobile sensor to the vicinity of the target. For this purpose, we propose to design a tracking controller for system (5) to follow system (1) with the main objective being the attenuation of the effect of the noise ω. The following control law is considered for the sensor system:

$$u_S = u_T + K\hat{z} = u_T + K(z - e), \tag{9}$$

where K is the controller gain, and we also use the nominal input of the target system, u_T. By using (9) we obtain the overall error system:

$$\begin{aligned} \dot{e} &= f_e(e, x_T, \hat{x}_T) - LD\omega(x_T, x_S) + Bd \\ \dot{z} &= f_z(z, x_T, x_S) + BKe + Bd \end{aligned} \tag{10}$$

$$f_e(e, x_T, \hat{x}_T) = (A - LC)e + G\left[\psi(Hx_T) - \psi(H\hat{x}_T)\right] \tag{11}$$

$$f_z(z, x_T, x_S) = (A - BK)z + G\left[\psi(Hx_T) - \psi(Hx_S)\right] \tag{12}$$

System (10) can be rewritten as

$$\dot{\chi} = \tilde{A}\chi + \tilde{G}\begin{bmatrix} \psi(Hx_T) - \psi(H\hat{x}_T) \\ \psi(Hx_T) - \psi(Hx_S) \end{bmatrix} - \begin{bmatrix} LD \\ 0 \end{bmatrix}\omega(x_T, x_S) + \tilde{B}d \tag{13}$$

where $\tilde{A} = \begin{bmatrix} A - LC & 0 \\ BK & A - BK \end{bmatrix}$, $\tilde{G} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} B \\ B \end{bmatrix}$, and $\chi = \begin{bmatrix} e^T & z^T \end{bmatrix}^T$. We denote $\eta = \begin{bmatrix} \eta_e^T & \eta_z^T \end{bmatrix}^T$, where

$$\eta_e := Hx_T - H\hat{x}_T = He, \quad \eta_z = Hx_T - Hx_S = Hz. \tag{14}$$

In view of (3), we can define $\delta_e(t) = \mathrm{diag}(\delta_{e1}(t), ..., \delta_{er}(t))$ and $\delta_z(t) = \mathrm{diag}(\delta_{z1}(t), ..., \delta_{zr}(t))$, to obtain

$$\begin{bmatrix} \psi(Hx_T) - \psi(H\hat{x}_T) \\ \psi(Hx_T) - \psi(Hx_S) \end{bmatrix} = \begin{bmatrix} \delta_e(t) & 0 \\ 0 & \delta_z(t) \end{bmatrix}\eta. \tag{15}$$

Denoting $\tilde{D} = \begin{bmatrix} -LD \\ 0 \end{bmatrix}$ and $\Delta(t) = \begin{bmatrix} \delta_e(t) & 0 \\ 0 & \delta_z(t) \end{bmatrix}$, we have

$$\dot{\chi} = f(\chi, \eta) + \tilde{D}\omega(x_T, x_S) + \tilde{B}d \tag{16}$$

where $f(\chi, \eta) = \tilde{A}\chi + \tilde{G}\Delta(t)\eta$.

Next, we will present three approaches to solve the observer and controller design problem. First, we use a monolithic approach for the augmented system presented in (10) without considering any disturbance on the input ($d = 0$). The disadvantage of this method is that the obtained conditions are defined as Bilinear Matrix Inequalities (BMI), which are hard to solve. In the second approach, we provide a more practical solution by decoupling the observer and controller design, and the results obtained will be combined to guarantee the stability of the augmented system (still for $d = 0$). Finally, we take into consideration the effect of the disturbance d, and sufficient conditions will be given to meet the H_∞ performance requirements.

III. Main results

A. Monolithic design

Our goal is to design the matrices L and K to ensure local stability for (16). To find these gains, Theorem 1 can be used.

Theorem 1: Consider system (16) with $d = 0$ and Assumption 2 holds. If matrices $\tilde{P} = \tilde{P}^T > 0$, $R_1 = R_1^T = \mathrm{diag}(r_{11}, ..., r_{1r}) > 0$, $R_2 = \mathrm{diag}(r_{21}, ..., r_{2r}) > 0$, L, K and constant ϵ can be found such that

$$\begin{bmatrix} \tilde{A}^T\tilde{P} + * + \epsilon I & \tilde{P}\tilde{G} + \tilde{H}^T R \\ * & \gamma(R) \end{bmatrix} \leq 0 \tag{17}$$

where $R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}$, $\tilde{H} = \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix}$, and $\gamma(R) = -2R\,\mathrm{diag}(\frac{1}{b_1}, ..., \frac{1}{b_r}, \frac{1}{b_1}, ..., \frac{1}{b_r})$, then (16) is locally asymptotically stable at the origin. □

Proof: Due to the lack of space, here we only give a sketch of the proof. The full proof is available at `lendek.net/files/papers/acc20-zoli-supp.pdf`.

First consider $\omega = 0$, let $V(\chi) := \chi^T\tilde{P}\chi$ for any $\chi \in \mathbb{R}^{2n}$. For any $\chi \in \mathbb{R}^{2n}$ and $\eta \in \mathbb{R}^{2r}$,

$$\langle\nabla V(\chi), f(\chi, \eta)\rangle = \begin{bmatrix} \chi \\ \Delta(t)\eta \end{bmatrix}^T \begin{bmatrix} \tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} & \tilde{P}\tilde{G} \\ * & 0 \end{bmatrix} \begin{bmatrix} \chi \\ \Delta(t)\eta \end{bmatrix}$$

By using (17), we have:

$$\begin{aligned} \langle\nabla V(\chi), f(\chi, \eta)\rangle &\leq \begin{bmatrix} \chi \\ \Delta(t)\eta \end{bmatrix}^T \begin{bmatrix} -\epsilon I & -\tilde{H}^T R \\ * & -\gamma(R) \end{bmatrix} \begin{bmatrix} \chi \\ \Delta(t)\eta \end{bmatrix} \\ &= -\epsilon\|\chi\|^2 - 2\chi^T\tilde{H}^T R\Delta(t)\eta - \eta^T\Delta(t)^T\gamma(R)\Delta(t)\eta. \end{aligned}$$

Using (14) we have $\chi^T\tilde{H}^T R\Delta(t)\eta = \eta_e^T R_1\delta_e(t)\eta_e + \eta_z^T R_2\delta_z(t)\eta_z$. This leads to

$$\begin{aligned} \langle\nabla V(\chi), f(\chi, \eta)\rangle &\leq -\|\chi\|^2 \\ &- 2\eta_e^T\left(R_1 - \delta_e^T R_1\mathrm{diag}\left[\tfrac{1}{b_1}, ..., \tfrac{1}{b_r}\right]\right)\delta_e(t)\eta_e \\ &- 2\eta_z^T\left(R_2 - \delta_z^T R_2\mathrm{diag}\left[\tfrac{1}{b_1}, ..., \tfrac{1}{b_r}\right]\right)\delta_z(t)\eta_z. \end{aligned}$$

R_1 and $\delta_e(t)$ are diagonal matrices, with diagonal terms: $r_{1k}\left(1 - \delta_{ek}\frac{1}{b_k}\right)$. Based on (3) we know that $\delta_{ek} \in [0, b_k]$, and $r_{1k} > 0$, which means that $r_{1k}(1 - \delta_{ek}\frac{1}{b_k}) \geq 0$, so

$$-2\eta_e^T\left(R_1 - \delta_e^T R_1\mathrm{diag}\left[\tfrac{1}{b_1}, ..., \tfrac{1}{b_r}\right]\right)\delta_e(t)\eta_e \leq 0, \tag{18}$$

A similar deduction holds for η_z, so $\langle\nabla V(\chi), f(\chi, \eta)\rangle \leq -\|\chi\|^2$, which proves global stability when $d = 0$, $\omega = 0$.

Next, we consider the uncertain term ω, and we prove local stability. The term ω can be considered as a vanishing perturbation and we can use Lemma 9.1 from [16]. We have:

$$\langle\nabla V(\chi), f(\chi, \eta) + \tilde{D}\omega(x_T, x_S)\rangle \leq -\epsilon\|\chi\|^2 + 2\chi^T\tilde{P}\tilde{D}\omega(x_T, x_S)$$

Based on Assumption 2, we obtain:

$$\omega(x_T, x_S) \leq \zeta_1\|z\|^{\zeta_2} \leq \zeta_1\|\chi\|^{\zeta_2},$$

which leads to

$$\begin{aligned} &\langle\nabla V(\chi), f(\chi, \eta) + \tilde{D}\omega(x_T, x_S)\rangle \leq \\ &\qquad - \|\chi\|^2\left(\epsilon - 2\zeta_1\|\tilde{P}\tilde{D}\|\|\chi\|^{\zeta_2 - 1}\right) \end{aligned}$$

■

Note that (17) is a BMI. To overcome this issue, next we propose a decoupled design.

B. Decoupled design

We give a decoupled approach in which we separately design the observer and controller gains, allowing us to write a set of LMIs instead of BMIs. The main advantage of this approach is that the LMIs can be efficiently solved by using optimization toolboxes. However, the design is more conservative compared to the monolithic approach.

Note that the observer design conditions from [17], [5], [6] are not suitable for controller design. Therefore, we first provide sufficient conditions for controller design.

Theorem 2: Consider the plant:

$$\begin{aligned}\dot{z} =&(A-BK)z + G\delta_z(t)\eta_z\\ \eta_z =&Hz\end{aligned} \tag{19}$$

where $\delta_z(t) = \mathrm{diag}(\delta_{z1}, ...\delta_{zr})$, and $\forall k \in [1, r]$, $\delta_{zk} \in [a_k, b_k], 0 \le a_k \le b_k < \infty$, and define $F = \mathrm{diag}(\frac{1}{b_1}, ..., \frac{1}{b_r})$. If there exist matrices $P = P^T > 0$, $Q = \mathrm{diag}(q_1, ..., q_r) > 0$, N, and a constant $\epsilon > 0$, so that

$$\begin{bmatrix} AP - BN + * & G(FQ)^T + PH^T & P \\ * & -2FQF & 0 \\ * & * & -\frac{1}{\epsilon}I \end{bmatrix} < 0 \tag{20}$$

then the origin of (19) is globally asymptotically stable, and the controller gain can be recovered from $K = NP^{-1}$. □

Proof: The Lyapunov function is $V(z) = z^T P^{-1} z$, and we impose that

$$\begin{aligned}&\langle \nabla V(z), (A - BK)z + G\delta_z(t)\eta_z \rangle < \\ &\begin{bmatrix} z \\ \delta_z(t)\eta_z \end{bmatrix}^T \begin{bmatrix} -\epsilon I & -H^T(FQ)^T \\ * & 2Q^{-1} \end{bmatrix} \begin{bmatrix} z \\ \delta_z(t)\eta_z \end{bmatrix}.\end{aligned} \tag{21}$$

Based on this, we have the following matrix inequality: $\begin{bmatrix} P^{-1}(A-BK)+*+\epsilon I & P^{-1}G+H^T(FQ)^{-T} \\ * & -2Q^{-1} \end{bmatrix} < 0$. Congruence with $\begin{bmatrix} P & 0 \\ 0 & FQ \end{bmatrix}$, Schur complement on ϵP^2 and denoting $M = KP$ gives (20). Since all F, Q and $\delta_z(t)$ are diagonal, by examining the terms we obtain that $2\eta_z^T \left((FQ)^{-T}\delta_z - \delta_z^T Q^{-1}\delta_z\right)\eta_z \le 0$. Finally, we have $\langle \nabla V(z), (A - BK)z + G\delta_z(t)\eta_z \rangle \le -\epsilon \|z\|^2 \le 0$ ■

Now we are ready to define the unperturbed ($d = 0$) decoupled design approach.

Theorem 3: Consider the plant (10) with $d = 0$ and Assumption 2 holds. If matrices $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $R_1 = R_1^T = \mathrm{diag}(r_{11}, ..., r_{1r}) > 0$, $Q = Q^T = \mathrm{diag}(q_1, ..., q_r) > 0$, M, N and constants ϵ_e and ϵ_z can be found such that:

$$\begin{bmatrix} A^T P_1 - C^T M^T + * + \epsilon_e I & P_1 G + H^T R_1 \\ * & \gamma(R_1) \end{bmatrix} \le 0$$

$$\begin{bmatrix} AP_2 - BN + * & G(FQ)^T + P_2 H^T & P_2 \\ * & -2FQF & 0 \\ * & * & -\frac{1}{\epsilon_z}I \end{bmatrix} \le 0,$$

where $F = \mathrm{diag}(\frac{1}{b_1}, ..., \frac{1}{b_r})$, and $\gamma(R_1) = -2R_1 F$, then the origin of (10) is locally asymptotically stable, and the observer and controller gains can be recovered from $L = P_1^{-1}M$ and $K = NP_2^{-1}$. □

Proof: Let, for any $e, z \in \mathbb{R}^n$ $V_e(e) := e^T P_1 e$, $V_z(z) := z^T P_2^{-1} z$, that satisfies the conditions of Theorems 1-2 with $\omega = 0$, we have: $\langle \nabla V_e(e), f_e(e, x_T, \hat{x}_T) \rangle \le -\epsilon_e \|e\|^2$ and $\langle \nabla V_z(z), f_z(z, x_T, x_S) \rangle \le -\epsilon_z \|z\|^2$. Now based on Section 7.6 from [3], there always exist constants λ, $\epsilon_{ez} > 0$ for $P = \begin{bmatrix} \lambda P_1 & 0 \\ 0 & P_2^{-1} \end{bmatrix}$, so that $\langle \nabla V(e, z), f(e, x_T, \hat{x}_T), f(z, x_T, x_S) + BKe \rangle \le -\epsilon_{ez} \|\chi\|^2$. Local stability is proved as in Theorem 1, when $\omega \ne 0$. ■

C. H_∞ design

In the first two approaches we considered the case when the input of the target system, u_T is known and we can use it both for observer and controller design. Next, we consider the scenario, when a nominal value of the target input, u_T, is known, but it is affected by an unknown disturbance term d.

Theorem 4: Consider system (16), if matrices $\tilde{P} = \tilde{P}^T > 0$, $R_1 = R_1^T = \mathrm{diag}(r_{11}, ..., r_{1r}) > 0$, $R_2 = \mathrm{diag}(r_{21}, ..., r_{2r}) > 0$, L, K, and constants ϵ, μ_d can be found such that

$$\begin{bmatrix} \tilde{A}^T \tilde{P} + * + \epsilon I & \tilde{P}\tilde{G} + \tilde{H}^T R & \tilde{P}\tilde{B} \\ * & \gamma(R) & 0 \\ * & * & -\mu_d I \end{bmatrix} \le 0 \tag{22}$$

where R, $\tilde{H}$, $\gamma(R)$ are as in Theorem 1, then the augmented error dynamics in (16) satisfies (23) for all $\|\chi\|^2 \le \beta$,

$$\langle \nabla V(\chi), f(\chi, \eta) + \tilde{D}\omega + \tilde{B}d \rangle - \mu_d \|d\|^2 \le 0. \tag{23}$$

Proof: Using (23), and based on Theorem 1 we have:

$$\begin{aligned}&\langle \nabla V(\chi), f(\chi, \eta) + \tilde{B}d \rangle - \mu_d \|d\|^2 \\ &\qquad \le -|\chi|^2(\epsilon - 2\zeta_1 \|\tilde{P}\tilde{D}\| \|\chi\|^{\zeta_2 - 1}).\end{aligned} \tag{24}$$

A constant $\|\chi\|^2 \le \beta$ exists, so $(\epsilon - 2\zeta_1 \|\tilde{P}\tilde{D}\| \|\chi\|^{\zeta_2 - 1}) \ge 0$, which leads to $\langle \nabla V(\chi), f(\chi, \eta) + \tilde{B}d \rangle - \mu_d \|d\|^2 \le 0$. For $\omega \ne 0$ we follow the steps in Theorem 1 to obtain (23). ■

D. Discussion on perturbation $\omega(x_T, x_S)$

The perturbation model in (4) is frequent in radio-frequency ranging measurements. For instance a model is given in [1] with zero mean and distance-dependent variance $\sigma^2 = \sigma_0^2 \left(\frac{w_i}{w_0}\right)^\alpha$, where σ_0^2 is the variance at reference distance w_0 and α is the path loss exponent. To apply this to our framework we approximate:
$w_i = \|Cx_T - Cx_S\| \le \|C\| \|x_T - x_S\|$, and take uniformly distributed noise in the finite interval $[-3\sigma, 3\sigma]$, which covers 95% of the probability mass of the original, normally-distributed noise. Therefore, perturbation model (4) applies with $\zeta_1 = \|C\|^{\alpha/2} \frac{3\sigma_0}{r_0^{\alpha}/2}$, $\zeta_2 = \frac{\alpha}{2}$, $\zeta_3 = 0$.

IV. Numerical example

We illustrate the results of Section III by considering two vehicles driving on a highway, as in [2]. The lead vehicle is the target, and its state must be estimated by the follower vehicle, which is the sensor. The vehicles can only move in a straight line, with the following model taken from [8]

$$\begin{aligned}\dot{x}_1 =& \; x_2 \\ \dot{x}_2 =& -c_1 - c_2 x_2^2 + c_3 u.\end{aligned} \tag{25}$$

Here, position x_1 is the system's output, and x_2 the velocity. We approximate the square nonlinearity for any $x_2 \in \mathbb{R}$ by

$$\psi_a(x_2) := \begin{cases} c_1 + c_2 x_2^2, & |x_2| \leq v_b \\ c_1 + c_2 v_b |x_2|, & |x_2| > v_b, \end{cases} \tag{26}$$

where v_b is a given velocity up to which the approximation is accurate. We take v_b equal to the highway speed limit, which means the approximation is accurate in typical driving regimes. Since we consider highway driving, the velocity is also taken to be positive, i.e. $v > 0$, in which case Assumption 1 is verified with $a = 0$ and $b = 2c_2 v_b$. We have

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ c_3 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \tag{27}$$
$$\psi(Hx) = \psi_a(x_2), \quad H = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Note that positive velocities are not enforced in the dynamics, instead they will be verified *a posteriori* in the simulation results.

As it was defined in Section II we consider the same structure for the target, $x_T = \begin{bmatrix} x_{T1} & x_{T2} \end{bmatrix}^T$, and for the mobile sensor, $x_S = \begin{bmatrix} x_{S1} & x_{S2} \end{bmatrix}^T$, with matrices defined in (27). For the output of the target system, we have $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = 1$. Based on [8], the parameters are $c_1 = 0.1$, $c_2 = 0.463$ and $c_3 = 3.6$. The threshold velocity is $v_b = 130[km/h] \simeq 37[m/s]$. We take the following parameter values: $\zeta_1 = 0.005$, $\zeta_2 = 2$. The parameter $\zeta_3 = 0.18$ defines a small distance in which the effect of the ranging noise is 0. In the first case we consider the unperturbed input for the target, $d = 0$. By using Theorem 3 the following gains were found:

$$L = \begin{bmatrix} 8.61 \\ 0.48 \end{bmatrix}, \quad K = \begin{bmatrix} 6.61 & 6.15 \end{bmatrix}, \tag{28}$$

using `sedumi` solver in the `Yalmip` framework. The maximum norm on the initial condition for the augmented states for which Theorem 3 provides local asymptotic stability at the origin is: $\|\chi_0\| \leq \rho = 43.86$. We choose the following initial conditions: $x_{T0} = [30,\, 0]^T$, $x_{S0} = [0,\, 0]^T$ and $\hat{x}_{T0} = [0,\, 0]^T$, so $\|\chi_0\| = 42.42$. A constant input is considered for the target system $u_T = 10$. The output of the target system

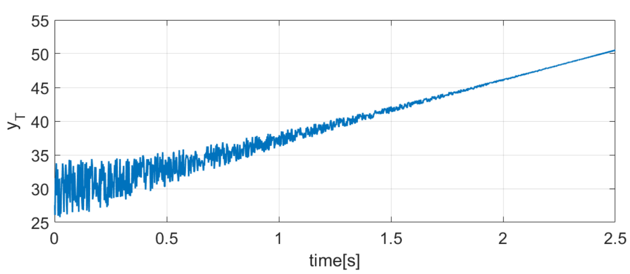

Fig. 1. Output y_T of the target, as seen by the sensor

can be seen in Fig 1. From 0 to 1.5 seconds, y is very noisy since the distance between the sensor and the target is large, so ω is large, but after that the effect of the ranging noise is attenuated. The results obtained for the estimation and tracking error are shown in Fig. 2. All the states of the error systems are converging to 0.

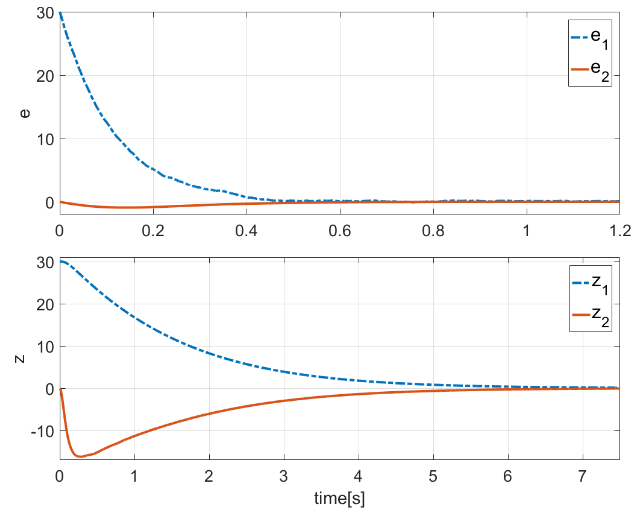

Fig. 2. Evolution of the estimation and tracking errors

In order to make the approach more realistic, we would like to maintain a safe distance between the sensor system and the target. For this particular example, such a goal can be achieved with a change of variables: $\tilde{z} := z - \begin{bmatrix} q \\ 0 \end{bmatrix}$, where $q > 0$ is a predefined reference distance that needs to be achieved and maintained between the two vehicles.

Note that $\dot{\tilde{z}}_1 = \dot{z}_1 = z_2$ and $\dot{z}_2$ depends only on z_2 and u_S. We define the following control law:

$$u_S = u + K\left(\hat{z} - \begin{bmatrix} q \\ 0 \end{bmatrix}\right) = u + K(\tilde{z} - e). \tag{29}$$

With this expression, our entire theoretical framework applies directly, with $\tilde{z}$ instead of z. Based on (4) and the parameter values ζ_1, ζ_2 and ζ_3, we find that if $\|x_T - x_S\| = \|z\| \leq 6$, then $\omega = 0$. We assume that the two vehicles will have almost the same velocity around this region, which leads to the conclusion that if the distance between the two vehicles is less then $6[m]$ the effect of the ranging noise is vanishing. Based on these we chose the tracking distance $q = 5[m]$. Simulation results can be seen on Fig. 3, from the same initial conditions as before. In this setup z_1 does not converge to

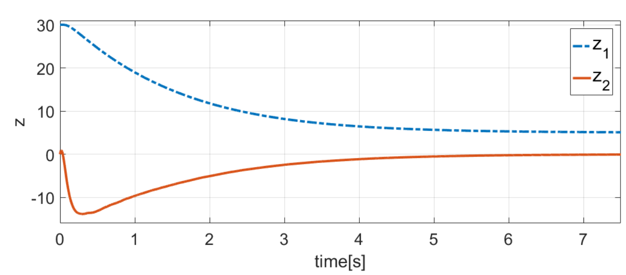

Fig. 3. Distance maintaining setup: $x_T - x_S$

0, but to q as we expected.

Finally, we consider the case of perturbed input, i.e., when $d \neq 0$ and verify the attenuation capabilities of the gains in (28). We define the LMI problem so that μ_d is minimized for condition (23). The obtained minimal value is $\mu_d = 0.23$, so the attenuation of the disturbance will be at least $\sqrt{\mu_d} = 0.48$. On the other hand, the maximum value of the initial $\|\chi\|$ decreased to $\|\chi_0\| = 23.61$. The new initial conditions according to this setup are the following: $x_{T0} = [16, 0]^T$, $x_{S0} = [0, 0]^T$ and $\hat{x}_{T0} = [0, 0]^T$, so $\|\chi_0\| = 22.62$. The input for the target system is $u_T = 10$, and we consider a constant disturbance $d = 2$ for 5 seconds and then $d = -1$. The obtained results are shown in Fig. 4, for both estimation and tracking error the effect of the input disturbance is attenuated.

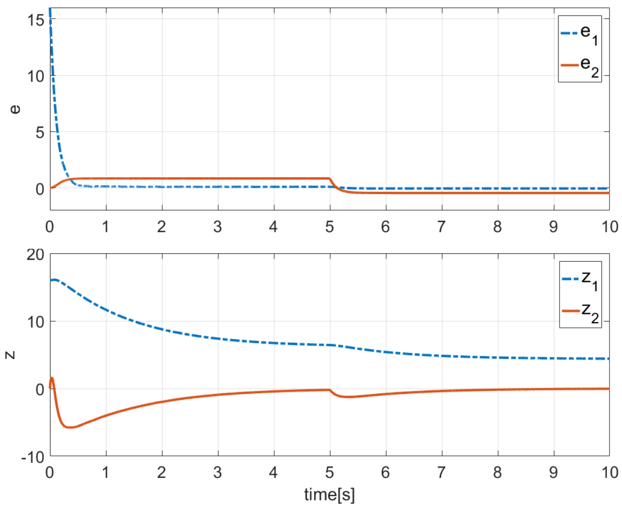

Fig. 4. Evolution of the estimation and tracking errors in presence of input disturbance

V. Conclusions

This paper presented a novel controller-based observer design approach for the scenario where a mobile sensor must monitor the state of a target system. In particular, the mobile sensor needs to be controlled so that a predefined distance from the target is achieved and maintained. We presented a case study in which a nonlinear target system was considered. First a monolithic approach was provided, in which the controller and observer design conditions are written as a single bilinear matrix inequality. Next, we provided a more practical approach, in which we decoupled the observer and controller design, and the problem was defined in terms of linear matrix inequalities. Finally, we considered the case when a nominal input of the target is known, but it is affected with an unknown disturbance term. In this final approach we defined sufficient conditions for which an H_∞ performance index was obtained for the attenuation of the disturbance. In order to study the effectiveness of these approaches, a numerical example was considered for two road vehicles in highway driving.

Among many future directions, we would like to extend the approach with more complex models that involve more than just scalar nonlinearities. On the other hand, we plan to investigate cases where the model of the sensor is different from the target.

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